

**Maximum Sustainable Withdrawal Rates With Varying Historical Success Rates  
Using Putnam Investors Fund, Intermediate-term Government Bonds and US T-Bills  
Ending Value Goal: Above Zero (Don't Go Broke)  
Data from 1962 to 2000  
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A rolling period analysis of historical investment performance is used to determine the maximum sustainable withdrawal rate and a portfolio capable of achieving that rate for various investment horizons and historical success rates. Every available time frame starting with 1962 ending 2000 is tested for each horizon (2, 3, ..., 30 years). Shorter horizons and higher success rates call for more conservative portfolios; longer horizons and lower success rates allow for more aggressive ones.

### **Methodology**

To understand the rolling period analysis, consider the 20-year horizon which has 20 observations (1962-1981, 1963-1982, ..., 1981-2000). Each of these observations is tested for each portfolio to determine historical success rate, defined as the percentage of time the portfolio is able to achieve the ending value for the account (in this case, to stay above \$0), given the initial withdrawal amount and withdrawal schedule.

The portfolio is rebalanced at the start of each year. Withdrawals occur at the start of the year. The Maximum Sustainable Withdrawal Rate (MSR) is the percentage of total account value withdrawn the first year. Subsequent withdrawals are adjusted by prior year inflation rates, maintaining buying power at the risk of running out of money.

For an example, consider the 20-year horizon from Table 2 (100% historical success rate): There are 20 observations (that is, time periods) to consider: 1962-1981, 1963-1982, ..., 1981-2000. The value for MSR of 5.17% means that there are one or more portfolios that, with 100% historical success, were able to survive 20 years of withdrawals from the account, starting with 5.17% of the value of the account the first year and adjusting each subsequent year's withdrawal by the prior year's inflation rate. Starting with one million dollars and the right asset allocation, you would have been able to withdraw \$51,700 the first year, then maintain buying power for the next 19 years without running out of money. This was true in every 20-year period.

Continuing the example, of the thousands of portfolios examined, the portfolio composed of 13.85% Putnam Investors Fund and 86.15% US T-Bills is the best performer. This portfolio achieves the goal of not running out of money in all 20 of the 20-year periods.

The example given uses a historical success rate of 100%. As you lower your historical success rate (and presumably your probability for future success) you gain higher levels of sustainable spending. Table 1 lists MSRs for all success rates but does not list asset allocations. Tables 2 through 9 show asset allocations with successively lower and lower levels of historical success, with the resulting higher and higher levels of MSR.

## Portfolio Optimizations

The WAT\$ Black Box program is used to find optimal portfolios. Black Box runs two processes, a systematic process and an optimization process. The systematic process creates portfolios by varying the percentage invested in each available asset by 5% at a time. So, for example, the following portfolios, as well as many others, are tested:

- 100% Putnam Investors Fund
- 65% Putnam Investors Fund, 35% US T-Bills
- 15% Putnam Investors Fund, 45% Intermediate-term Government Bonds, 40% US T-Bills

During the optimization process, the best portfolios from the systematic testing are kept and refined in an attempt to find better portfolios. This is done through a process known as simulated annealing, in which many more portfolios are tested to see if better portfolios exist. The idea of annealing comes from metallurgy: as molten metal cools the atoms gradually move less and less, eventually coming to a stable state. By repeatedly shrinking our test areas around known good portfolios we are able to gradually come to better and better portfolios.

The method of simulated annealing does not guarantee we will find the optimal portfolio, but it does find very good portfolios. If, for example, the WAT\$ Black Box program lists the optimal portfolio as being capable of a 5% withdrawal rate, it is very unlikely there exists a portfolio (using the same constraints and assets) that can achieve a 5.1% withdrawal rate.

There is an anomaly in the MSR in that longer horizons sometimes show higher withdrawal rates than shorter horizons. The anomaly is a result of the rolling period technique used. Each time a year is added to the horizon we lose one observation. The correct interpretation is to extend the lowest withdrawal rate in each column through the end of the table, so that, in fact, the maximum sustainable withdrawal rate for the longer horizons is the lowest value in the column. We have highlighted the text in the tables to indicate these anomalies.

*In this study the assets available to the portfolio are restricted to Putnam Investors Fund, Intermediate-term Government Bonds and US T-Bills.*

*All data from Ibbotson Associates, Inc. except return data for Putnam Investors Fund which comes from S&P Micropal.*

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*Study conducted by Zunna, Inc. using the WAT\$ Black Box Program.*

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**Table 1 - Maximum Sustainable Withdrawal Rates  
With Varying Historical Success Rates  
Ending Value Goal: Above Zero (Don't Go Broke)  
1962-2000**

Horizon Years	# Obs	Historical Success Rate							
		100%	95%	90%	85%	80%	75%	66%	50%
2	38	49.04	49.41	49.66	49.98	50.11	50.19	50.76	52.60
3	37	32.36	32.52	33.09	33.67	33.94	34.35	34.87	35.76
4	36	24.13	24.18	24.61	25.18	26.03	26.66	27.27	27.91
5	35	19.19	19.23	19.99	20.64	21.09	21.66	22.28	23.12
6	34	15.89	15.92	16.84	17.39	17.68	18.23	18.87	19.84
7	33	13.51	13.52	14.46	14.72	15.21	15.82	16.45	17.61
8	32	11.72	11.73	12.56	12.97	13.39	13.79	14.87	15.86
9	31	10.34	10.43	11.14	11.41	11.94	12.15	13.62	14.40
10	30	9.29	9.43	9.97	10.25	10.75	11.07	12.55	13.41
11	29	8.49	8.58	8.69	9.27	9.43	10.35	11.10	12.49
12	28	7.84	7.94	8.08	8.55	8.64	9.81	10.45	11.39
13	27	7.30	7.43	7.61	7.90	7.92	8.65	9.62	10.73
14	26	6.85	6.99	7.11	7.23	7.47	8.21	8.52	10.31
15	25	6.48	6.65	6.72	6.89	7.10	7.79	7.95	9.29
16	24	6.14	6.37	6.39	6.59	6.61	7.44	7.52	9.02
17	23	5.85	6.09	6.11	6.32	6.37	6.50	7.09	8.08
18	22	5.59	5.86	5.89	6.05	6.15	6.29	6.75	7.75
19	21	5.37	5.67	5.71	5.85	5.98	6.09	6.51	7.07
20	20	5.17	5.49	5.54	5.68	5.83	5.91	6.18	6.85
21	19	5.00	5.00	5.33	5.40	5.52	5.70	5.93	6.48
22	18	4.85	4.85	5.20	5.28	5.38	5.58	5.75	6.35
23	17	4.71	4.71	5.08	5.18	5.27	5.49	5.54	6.15
24	16	4.60	4.60	4.98	5.07	5.16	5.41	5.44	6.01
25	15	4.50	4.50	4.89	5.00	5.07	5.07	5.35	5.57
26	14	4.43	4.43	4.81	4.94	4.94	4.99	5.22	5.47
27	13	4.37	4.37	4.74	4.74	4.89	4.92	5.12	5.29
28	12	4.32	4.32	4.68	4.68	4.85	4.85	5.03	5.32
29	11	4.63	4.63	4.81	4.81	4.83	4.83	4.96	5.25
30	10	4.60	4.60	4.76	4.76	4.89	4.89	5.10	5.47

Assets Available to the Portfolio: Putnam Investors Fund (PINVX), Intermediate-term Government Bonds (IG) and US T-Bills (T)  
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**Table 2 - Maximum Sustainable Withdrawal Rates  
With Historical Success Rate of 100%  
Ending Value Goal: Above Zero (Don't Go Broke)  
1962-2000**

Horizon Years	#Obs	MSR	Portfolio		
			PINVX	IG	T
2	38	49.04			100.00
3	37	32.36			100.00
4	36	24.13			100.00
5	35	19.19			100.00
6	34	15.89			100.00
7	33	13.51			100.00
8	32	11.72			100.00
9	31	10.34			100.00
10	30	9.29			100.00
11	29	8.49			100.00
12	28	7.84			100.00
13	27	7.30	0.88		99.12
14	26	6.85	1.90		98.10
15	25	6.48	3.07		96.93
16	24	6.14	3.37		96.63
17	23	5.85	5.45		94.54
18	22	5.59	7.44	0.02	92.55
19	21	5.37	9.16		90.84
20	20	5.17	13.85		86.15
21	19	5.00	20.00		80.00
22	18	4.85	25.00		75.00
23	17	4.71	30.61		69.39
24	16	4.60	35.00		65.00
25	15	4.50	40.00		60.00
26	14	4.43	45.00		55.00
27	13	4.37	48.76		51.24
28	12	4.32	51.44		48.56
29	11	4.63	91.95		8.04
30	10	4.60	93.34		6.66

Assets Available to the Portfolio: Putnam Investors Fund (PINVX), Intermediate-term Government Bonds (IG) and US T-Bills (T)  
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**Table 3 - Maximum Sustainable Withdrawal Rates  
With Historical Success Rate of 95%  
Ending Value Goal: Above Zero (Don't Go Broke)  
1962-2000**

Horizon Years	#Obs	MSR	Portfolio		
			PINVX	IG	T
2	38	49.41	3.24	0.01	96.75
3	37	32.52			100.00
4	36	24.18		3.05	96.95
5	35	19.23		2.11	97.89
6	34	15.92		5.83	94.17
7	33	13.52			100.00
8	32	11.73			100.00
9	31	10.43			100.00
10	30	9.43	15.00		85.00
11	29	8.58	20.00		80.00
12	28	7.94	28.88		71.12
13	27	7.43	35.25		64.75
14	26	6.99	39.58		60.42
15	25	6.65	45.00		55.00
16	24	6.37	50.00		50.00
17	23	6.09	56.97		43.03
18	22	5.86	60.00		40.00
19	21	5.67	65.00		35.00
20	20	5.49	68.16		31.84
21	19	5.00	20.00		80.00
22	18	4.85	25.00		75.00
23	17	4.71	30.00		70.00
24	16	4.60	35.00		65.00
25	15	4.50	40.00		60.00
26	14	4.43	45.00		55.00
27	13	4.37	47.91		52.09
28	12	4.32	52.43	0.03	47.53
29	11	4.63	91.23		8.77
30	10	4.60	93.67	0.01	6.32

Assets Available to the Portfolio: Putnam Investors Fund (PINVX), Intermediate-term Government Bonds (IG) and US T-Bills (T)  
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**Table 4 - Maximum Sustainable Withdrawal Rates  
With Historical Success Rate of 90%  
Ending Value Goal: Above Zero (Don't Go Broke)  
1962-2000**

Horizon Years	#Obs	MSR	Portfolio		
			PINVX	IG	T
2	38	49.66	14.04		85.96
3	37	33.09	18.34		81.66
4	36	24.61	7.93		92.07
5	35	19.99	61.03		38.97
6	34	16.84	64.75		35.25
7	33	14.46	31.07		68.93
8	32	12.56	20.00		80.00
9	31	11.14	25.00		75.00
10	30	9.97	23.69		76.30
11	29	8.69	18.42		81.58
12	28	8.08	23.66		76.34
13	27	7.61	32.87		67.13
14	26	7.11	45.00		55.00
15	25	6.72	50.00		50.00
16	24	6.39	55.00		45.00
17	23	6.11	53.92		46.08
18	22	5.89	58.00		42.00
19	21	5.71	61.83		38.17
20	20	5.54	65.00		35.00
21	19	5.33	70.00		30.00
22	18	5.20	74.80		25.20
23	17	5.08	77.17		22.83
24	16	4.98	79.21		20.79
25	15	4.89	82.02		17.98
26	14	4.81	85.00		15.00
27	13	4.74	87.23		12.77
28	12	4.68	90.00		10.00
29	11	4.81	100.00		
30	10	4.76	100.00		

Assets Available to the Portfolio: Putnam Investors Fund (PINVX), Intermediate-term Government Bonds (IG) and US T-Bills (T)  
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**Table 5 - Maximum Sustainable Withdrawal Rates  
With Historical Success Rate of 85%  
Ending Value Goal: Above Zero (Don't Go Broke)  
1962-2000**

Horizon Years	#Obs	MSR	Portfolio		
			PINVX	IG	T
2	38	49.98	7.60		92.40
3	37	33.67	43.39		56.61
4	36	25.18	22.29		77.71
5	35	20.64	78.83		21.17
6	34	17.39	90.00		10.00
7	33	14.72	68.60	6.82	24.58
8	32	12.97	65.25	15.76	18.99
9	31	11.41	57.73	11.39	30.88
10	30	10.25	56.14	14.96	28.91
11	29	9.27	56.89	13.20	29.91
12	28	8.55	65.59	1.61	32.79
13	27	7.90	63.54		36.46
14	26	7.23	40.00		60.00
15	25	6.89	46.58		53.42
16	24	6.59	60.69		39.31
17	23	6.32	78.90		21.10
18	22	6.05	80.00		20.00
19	21	5.85	83.24		16.76
20	20	5.68	85.00		15.00
21	19	5.40	68.65		31.35
22	18	5.28	72.18		27.82
23	17	5.18	75.00		25.00
24	16	5.07	78.98		21.02
25	15	5.00	80.91		19.09
26	14	4.94	84.38		15.62
27	13	4.74	86.38		13.62
28	12	4.68	90.00		10.00
29	11	4.81	100.00		
30	10	4.76	100.00		

Assets Available to the Portfolio: Putnam Investors Fund (PINVX), Intermediate-term Government Bonds (IG) and US T-Bills (T)  
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**Table 6 - Maximum Sustainable Withdrawal Rates  
With Historical Success Rate of 80%  
Ending Value Goal: Above Zero (Don't Go Broke)  
1962-2000**

Horizon Years	#Obs	MSR	Portfolio		
			PINVX	IG	T
2	38	50.11	5.10		94.90
3	37	33.94	59.49	3.61	36.90
4	36	26.03	58.97	16.30	24.74
5	35	21.09	70.92	6.86	22.23
6	34	17.68	74.96	25.04	
7	33	15.21	100.00		
8	32	13.39	66.66	0.04	33.30
9	31	11.94	85.81	14.05	0.13
10	30	10.75	90.88		9.12
11	29	9.43	53.22	46.78	
12	28	8.64	53.61	46.39	
13	27	7.92	66.76	14.17	19.07
14	26	7.47	72.93		27.07
15	25	7.10	76.64		23.36
16	24	6.61	75.00		25.00
17	23	6.37	57.24		42.75
18	22	6.15	62.62		37.38
19	21	5.98	67.54	0.01	32.45
20	20	5.83	71.21		28.79
21	19	5.52	89.13		10.87
22	18	5.38	90.68		9.32
23	17	5.27	92.86		7.14
24	16	5.16	95.00		5.00
25	15	5.07	96.83		3.17
26	14	4.94	83.97		16.03
27	13	4.89	86.66		13.34
28	12	4.85	94.44		5.56
29	11	4.83	90.00		10.00
30	10	4.89	100.00		

Assets Available to the Portfolio: Putnam Investors Fund (PINVX), Intermediate-term Government Bonds (IG) and US T-Bills (T)  
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**Table 7 - Maximum Sustainable Withdrawal Rates  
With Historical Success Rate of 75%  
Ending Value Goal: Above Zero (Don't Go Broke)  
1962-2000**

Horizon Years	#Obs	MSR	Portfolio		
			PINVX	IG	T
2	38	50.19	3.11	1.65	95.24
3	37	34.35	98.97		1.03
4	36	26.66	100.00		
5	35	21.66	100.00		
6	34	18.23	100.00		
7	33	15.82	100.00		
8	32	13.79	100.00		
9	31	12.15	75.00		25.00
10	30	11.07	82.73		17.26
11	29	10.35	100.00		
12	28	9.81	100.00		
13	27	8.65	100.00		
14	26	8.21	100.00		
15	25	7.79	100.00		
16	24	7.44	100.00		
17	23	6.50	80.58		19.42
18	22	6.29	83.57		16.42
19	21	6.09	86.72	0.02	13.25
20	20	5.91	88.67	0.15	11.18
21	19	5.70	75.00		25.00
22	18	5.58	78.85		21.14
23	17	5.49	82.56	0.02	17.42
24	16	5.41	89.44		10.56
25	15	5.07	96.67		3.33
26	14	4.99	99.47		0.53
27	13	4.92	100.00		
28	12	4.85	100.00		
29	11	4.83	90.00		10.00
30	10	4.89	100.00		

Assets Available to the Portfolio: Putnam Investors Fund (PINVX), Intermediate-term Government Bonds (IG) and US T-Bills (T)  
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**Table 8 - Maximum Sustainable Withdrawal Rates  
With Historical Success Rate of 66%  
Ending Value Goal: Above Zero (Don't Go Broke)  
1962-2000**

Horizon Years	#Obs	MSR	Portfolio		
			PINVX	IG	T
2	38	50.76	91.16	8.84	
3	37	34.87	99.19	0.81	
4	36	27.27	100.00		
5	35	22.28	100.00		
6	34	18.87	99.17	0.83	
7	33	16.45	100.00		
8	32	14.87	100.00		
9	31	13.62	100.00		
10	30	12.55	97.89	2.09	0.02
11	29	11.10	100.00		
12	28	10.45	100.00		
13	27	9.62	100.00		
14	26	8.52	100.00		
15	25	7.95	100.00		
16	24	7.52	100.00		
17	23	7.09	98.55		1.45
18	22	6.75	100.00		
19	21	6.51	100.00		
20	20	6.18	100.00		
21	19	5.93	100.00		
22	18	5.75	100.00		
23	17	5.54	95.69	4.26	0.05
24	16	5.44	100.00		
25	15	5.35	92.79	2.98	4.24
26	14	5.22	100.00		
27	13	5.12	100.00		
28	12	5.03	100.00		
29	11	4.96	100.00		
30	10	5.10	100.00		

Assets Available to the Portfolio: Putnam Investors Fund (PINVX), Intermediate-term Government Bonds (IG) and US T-Bills (T)  
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**Table 9 - Maximum Sustainable Withdrawal Rates  
With Historical Success Rate of 50%  
Ending Value Goal: Above Zero (Don't Go Broke)  
1962-2000**

Horizon Years	#Obs	MSR	Portfolio		
			PINVX	IG	T
2	38	52.60	100.00		
3	37	35.76	100.00		
4	36	27.91	100.00		
5	35	23.12	99.38	0.62	
6	34	19.84	88.42	11.58	
7	33	17.61	100.00		
8	32	15.86	100.00		
9	31	14.40	100.00		
10	30	13.41	100.00		
11	29	12.49	100.00		
12	28	11.39	88.63	11.37	
13	27	10.73	98.08	1.92	
14	26	10.31	99.61	0.39	
15	25	9.29	100.00		
16	24	9.02	100.00		
17	23	8.08	100.00		
18	22	7.75	100.00		
19	21	7.07	100.00		
20	20	6.85	100.00		
21	19	6.48	100.00		
22	18	6.35	100.00		
23	17	6.15	100.00		
24	16	6.01	100.00		
25	15	5.57	100.00		
26	14	5.47	100.00		
27	13	5.29	92.76		7.24
28	12	5.32	100.00		
29	11	5.25	100.00		
30	10	5.47	100.00		

Assets Available to the Portfolio: Putnam Investors Fund (PINVX), Intermediate-term Government Bonds (IG) and US T-Bills (T)  
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